

## **Book Review: *Theory and Applications of Long-Range Dependence***

**Theory and Applications of Long-Range Dependence.** Paul Doukhan, George Oppenheim, and Murad Taqqu, Birkhäuser Press, Boston, 2003.

The phrase long-range dependence (LRD), also called long memory or strong dependence, is used to describe a time series with slowly decaying correlations. Mandelbrot coined the term “Joseph effect” in the 1960’s to describe the tendency of some time series to have persistent trends or cycles, after the Old Testament Joseph who predicted seven years of abundance followed by seven lean years. With the increasing number of large data sets being collected, there are a number of examples that exhibit LRD, e.g., Hurst’s analysis of historic measurements of the height of the Nile River, Mandelbrot’s studies of financial data, recent analysis of internet traffic, turbulence, the formation of cell structures for matter in the universe, etc.

This large volume (719 pages) is divided into four parts: probability theory (11 chapters), statistical theory (7 chapters), applications (7 chapters), and practical methodologies (4 chapters). A short description of the contents follows.

The probability section starts with a nice tutorial on fractional Brownian motion (fBm) by M. Taqqu. It summarizes many of the basic facts about fBm, fractional Gaussian noise, self-similarity (where  $X(at)$  has the same distribution as  $a^H X(t)$ ,  $H$  = Hurst exponent) and central limit theorems (CLT), and other facts. Other chapters are on historical comments on fBm, limit theorems for stationary sequences, seasonal long-memory, CLTs for polynomial forms, results on U-statistics, limit theorems for infinite variance sequences (stable and semi-stable laws), fractional calculus and fBm, and stochastic integration w.r.t. fBm.

The statistical section discusses estimation under long-range dependence assumptions: parametric estimates of  $H$ , semi-parametric spectral estimation, non-parametric density and regression problems, volatility in

financial time series, change-points/changes in regime, robust regression, and prediction for long-memory time series.

The application section starts with survey papers on LRD and data network traffic, on queues with LRD inputs, and on the LRD paradigm in finance. A chapter on LRD and ARCH modelling argues that observed financial times series have very good mixing properties, which is at odds with long-range dependence for stationary series; the authors argue that non-stationarity can explain both these observed features. The section ends with chapters on LRD in hydrology, turbulence in atmospheric temperature, and limit theorems for Burger's turbulence.

Finally, the methodology section has four longer survey chapters on using wavelets to analyze self-similarity and LRD, semi-parametric estimation of the LRD parameter, methods for simulating LRD series, and multifractal processes.

This book is a valuable and unique collection of material on LRD. It gathers together a large amount of material; the papers and their extensive references here give a broad and current summary of the research on the topic of long-range dependence.

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